

Practice Test D: Limits, Continuity, IVT

Name: _____

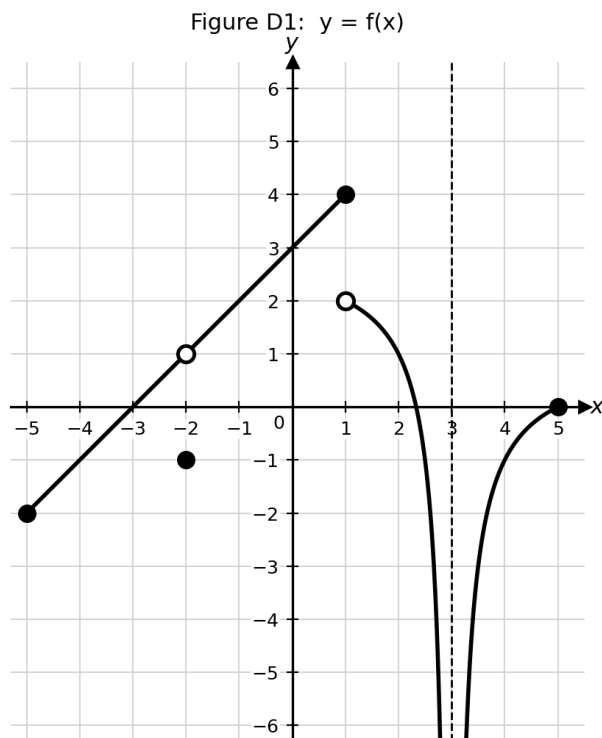
Time: 60 minutes No calculator 100 points

Answer exactly the question asked. Show work on every problem in Parts III–V.

If a limit is infinite, write ∞ or $-\infty$. If a limit does not exist for any other reason, write DNE.

If a function value does not exist, write *undefined*.

Part I. Graph. Questions 1(a)–1(g) refer to Figure D1.



1. Use the figure to find each value.

(a) $\lim_{x \rightarrow 3^+} f(x)$ _____

(b) $\lim_{x \rightarrow 3} f(x)$ _____

(c) $f(3)$ _____

(d) $\lim_{x \rightarrow 1^+} f(x)$ _____

(e) $\lim_{x \rightarrow 1} f(x)$ _____

(f) $f(-2)$ _____

(g) $\lim_{x \rightarrow -2} f(x)$ _____

(2 pts each, 14 pts)

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Part II. Table.

2. The table gives values of m . Assume the pattern continues as x approaches 2. **(2 each, 6 pts)**

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$m(x)$	-10	-100	-1000	0	1000	100	10

- (a) $\lim_{x \rightarrow 2^-} m(x)$ _____ (b) $\lim_{x \rightarrow 2} m(x)$ _____ (c) $m(2)$ _____

Part III. Algebra. Show work.

3. Evaluate $\lim_{t \rightarrow 2} \frac{t^2 - 9}{t^2 + t - 12}$, if it exists. **(5 pts)**

4. Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x+1}$, if it exists. **(5 pts)**

5. Evaluate $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 + 5x + 6}$, if it exists. **(6 pts)**

6. Let $g(x) = \frac{x+1}{(x-4)^2}$. Evaluate each limit. **(3 each, 9 pts)**

- (a) $\lim_{x \rightarrow 4^-} g(x)$ (b) $\lim_{x \rightarrow 4^+} g(x)$ (c) $\lim_{x \rightarrow 4} g(x)$

7. Let $q(t) = \begin{cases} 3t + 1 & t \leq -1 \\ t^2 + 1 & t > -1 \end{cases}$ (2 each, 10 pts)

- (a) $\lim_{t \rightarrow -3} q(t)$ (b) $\lim_{t \rightarrow -1^-} q(t)$ (c) $\lim_{t \rightarrow -1^+} q(t)$ (d) $\lim_{t \rightarrow -1} q(t)$ (e) $q(-1)$

8. Let $v(x) = \frac{x^3 + 1}{x + 1}$. Find $v(-1)$, if it is defined. (3 pts)

Part IV. Continuity. For 9 and 10, check each of the three conditions in the definition of continuity at a point and state your conclusion.

9. Using Figure D1, determine whether f is continuous at $x = 0$. (6 pts)

10. Let $k(x) = \begin{cases} x^2 - x - 3 & x \leq 3 \\ 2x - 4 & x > 3 \end{cases}$

Determine whether k is continuous at $x = 3$. (6 pts)

11. Using Figure D1, is f continuous on the closed interval $[-1, 1]$? Explain. (4 pts)

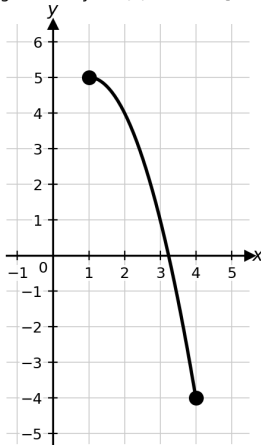
12. Let $u(x) = \frac{x^2 - 1}{2x^2 - 7x + 3}$. (3 each, 6 pts)
- (a) Is u continuous on $[0, 1]$? Explain. (b) Is u continuous on $(3, 5]$? Explain.

Part V. Intermediate Value Theorem.

13. The function $h(x) = x^3 - 2x^2 + 5$ is continuous on $[-1, 2]$. Find $h(-1)$. (2 pts)
14. Let $f(x) = x^2 - 6x$ on the interval $[2, 7]$. Does the Intermediate Value Theorem guarantee a number c in $[2, 7]$ with $f(c) = -5$? If it does, find every such c in $[2, 7]$. (6 pts)

15. Figure D2 shows the graph of s on its domain $[1, 4]$. Does the Intermediate Value Theorem guarantee a number c in $(1, 4)$ with $s(c) = 1$? If it does, find c . Explain. (4 pts)

Figure D2: $y = s(x)$, domain $[1, 4]$



16. True or False. Circle one. (4 each, 8 pts)
- (a) **T** **F** If f is continuous on $[2, 6]$, $f(2) = 4$, and $f(6) = 4$, then there is no number c in $(2, 6)$ with $f(c) = 10$.
- (b) **T** **F** If g is continuous on $[-1, 1]$, $g(-1) = -2$, and $g(1) = 3$, then there is exactly one number c in $[-1, 1]$ with $g(c) = 0$.